6.2 - Unit Circle and the six trigonometric functions

Recall a circle of radius, \( r \), centered at the origin \((0,0)\) has an equation written as:

\[
\text{Circumference of a circle of radius, } r, \text{ is} \]

\[
C = \]

For a UNIT CIRCLE, we must have a radius, \( r \), of 1 unit (that is \( r = 1 \))

\[
\text{Circumference of the unit circle is:} \]

\[
C = \]

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Imagine placing one end of a retractable string at the point \( P = (1,0) \) on the unit circle in the \( xy \)-plane. Let \( t \) be any real number where \( t = 0 \) is at \( P = (1,0) \) and for \( t > 0 \) (string expands counterclockwise) and \( t < 0 \) (string expands clockwise).

- Pull the string to travel around the unit circle clockwise to \((0,1)\) --- the distance it has traveled is \( t = \)
- Pull the string to travel around the unit circle clockwise to \((-1,0)\) --- the distance it has traveled is \( t = \)
- Pull the string to travel around the unit circle clockwise to \((0,-1)\) --- the distance it has traveled is \( t = \)
- Pull the string to travel around the unit circle clockwise to \((1,0)\) --- the distance it has traveled is \( t = \)

Indicate the point on the unit circle below where: (A) traveled \( t = \frac{2\pi}{3} \) and (B) traveled \( t = -\frac{3\pi}{4} \).
No matter what real number $t$ we choose, there is a unique point $P = (x, y)$ on the unit circle. The coordinates of the point $P = (x, y)$ on the unit circle corresponding to the real number $t$ allow us to define six trigonometric functions:

1. Sine function
2. Cosine function
3. Tangent function
4. Cosecant function
5. Secant function
6. Cotangent function
Determine if the point $P$ lives on the unit circle.

(a) $P = \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$
(b) $P = \left( \frac{2}{5}, \frac{3}{5} \right)$
(c) $P = \left( -\frac{\sqrt{5}}{3}, \frac{2}{3} \right)$

Let $P = \left( -\frac{\sqrt{5}}{3}, \frac{2}{3} \right)$ be a point on the unit circle that corresponds to a real number $t$.

Find the 6 trigonometric functions based on the point $P$. 
Let \( t \) be any real number and \( P = (x, y) \) is any point on the unit circle corresponding to \( t \).

\( \theta \) is an angle in standard position measured in RADIANS whose terminal side if the ray from the origin thru \( P = (x, y) \).

*** Since this is on the unit circle where \( r = 1 \) we must have that

\[
\begin{align*}
\sin(\theta) &= \\
\cos(\theta) &= \\
\tan(\theta) &= \\
\csc(\theta) &= \\
\sec(\theta) &= \\
\cot(\theta) &=
\end{align*}
\]

To find the EXACT value of a trigonometric function of an angle \( \theta \) (or real number \( t \)), we need to find the point \( P = (x, y) \) on the unit circle where the ray intersects the unit circle. There are a few NICE ANGLES that this can be done easily for.

Find the exact value of the SIX trigonometric function where \( \theta = \pi \)
6.2 - cont.

Find the exact value of the SIX trigonometric functions for \( \theta = \frac{\pi}{4} = 45^\circ \)

Find the exact value the following trigonometric function where \( \theta = \frac{5\pi}{4} \)
Find the exact value of the SIX trigonometric functions for \( \theta = \frac{\pi}{6} \) and \( \theta = \frac{\pi}{3} \).

What is the point \( P = (x, y) \) where \( \theta = \frac{2\pi}{3} \)?
Evaluate the following trigonometric function based on the given angle:

(a) \[ \tan \left( \frac{4\pi}{3} \right) = \]

(b) \[ \sec \left( \frac{11\pi}{6} \right) = \]

(c) \[ \sin \left( 60^\circ \right) - \cos \left( 150^\circ \right) = \]
6.2 - Extension of unit circle to any circle of radius \( r \)

Noticing the similar triangles we can justify that the ratios of corresponding sides are **EQUAL**.

That is,

\[
x^2 + y^2 = r^2
\]

**Example:** Let the point \( P = (-2, 5) \) be on the terminal side of an angle \( \theta \) in standard position. Find the exact values of the six trigonometric functions.