Math 141 Final (Sample)

Name:

1. Answer the following;

(a) What is the domain of $f(x) = \dfrac{x}{x^2 - 1}$.

(b) A relation is given $\{(1, c), (2, e), (3, a), (4, m), (5, k)\}$. Is it a function? If so, what are the domain and the range?

(c) A relation is given $\{(1, 2), (2, 3), (3, 2), (3, 0), (4, 8), (4, 3)\}$. Is it a function? If so, what are the domain and the range?

2. Find the linear function $y = f(x)$ whose graph passes through points $(2, 6)$ and $(4, 10)$.

3. Suppose $f(x) = 2x - 1$ and $g(x) = x^2 - 3$.
   (a) Find $(f \circ g)(x)$.

   (b) Find $(g \circ f)(x)$

4. The graph of a function f is given as below.

(See Figure 21 on Page 73)

Find the following;

  (a) The intercepts, if any.

  (b) The domain and range.

  (c) The interval on which it is increasing, decreasing, or constant.

  (d) The local extrema, if any.

  (e) The absolute maximum and minimum.

  (f) Sketch the graph of $F(x) = -f(x)$.

  (g) Sketch the graph of $G(x) = f(-x)$.

  (h) Sketch the graph of $H(x) = f(x+1)$.

5. $f(x)$ is a function defined by $f(x) = \dfrac{x^4}{x^2+3}$.

(a) Calculate $f(-x)$.

(b) Calculate $-f(x)$.

(c) Determine algebraically whether f is even or odd.

6. (a) Jae has 800 yards of fencing available to enclose a rectangular field. Express the area A of the rectangle as a function of the width x of the rectangle.

(b) An open box with a square base is to be made from a square piece of cardboard 60 inches on a side by cutting out a square from each corner and turning up the sides. Express the volume V of the box as a function of the length x of the side of the square cut from each corner.

7. (a) Find the x-intercepts of $y = f(x) = x^2 + x + 2$, if any. Does the graph open up or down? Using the graph of f, solve the inequality $x^2 + x + 2 \geq 0$.

(b) Find the x-intercepts of $y = g(x) = -x^2 - x + 12$, if any. Does the graph open up or down? Using the graph of f, solve the inequality $-x^2 - x + 12 \geq 0$.

8. (a) Find the quotient and the remainder when $3x^3 + 2x^2 + 6x + 2$ is divided by $x^2 - 3x + 2$.

(b) Find the vertical, horizontal, and oblique asymptotes, if any, of

$$R(x) = \frac{3x^3 + 2x^2 + 6x + 2}{x^2 - 3x + 2}$$

9. For $f(x) = (x+2)^2 (x-1)^3 (x+1)$, do the following.

(a) List each real zero and its multiplicity.

(b) Determine whether the graph crosses or touches the x-axis at each x-intercept.

(c) Determine the maximum number of turning points on the graph.

(d) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

(e) Sketch the graph of f.

(f) Solve the inequality $(x+2)^2 (x-1)^3 (x+1) < 0$.

10. Let $R(x) = \dfrac{x^2 + 4x + 3}{x^2 - x - 2}$. Answer the following.

    (a) Find the domain.

    (b) Write $H(x)$ in lowest terms.

    (c) Find the intercept(s) of the graph.

    (d) Find the vertical asymptote(s).

    (e) Find the horizontal or oblique asymptote, if one exist.

    (f) Sketch the graph.

    (g) Use the graph to solve the inequality $\dfrac{x^2 + 4x + 3}{x^2 - x - 2} > 0$.

11. (a) The following is the graph of a function $y = f(x)$.

Draw the graph of $y = f^{-1}(x)$

(b) Find the inverse of $y = g(x) = \dfrac{1}{x+1}$.

12. (a) Solve the equation $\log_2 x + \log_2 (x-1) = 1$.

(b) Solve the equation $3^{2x+1} = 5^{x-1}$.

13. (a) Write $\log_3\left[\dfrac{x^4\sqrt{x+1}}{(x-2)^3}\right]$ as a sum and/or difference of logarithms. Express powers as factors.

(b) Write $\log_5 7 + 2\log_5\left(\dfrac{x+3}{x+1}\right) + \log_5(x+1)$ as a single logarithm.

14. Let $f(x) = \log_2(x-3)$.

(a) Find the domain.

(b) Graph f.

(c) From the graph, determine the range and vertical asymptote of f.

(d) Find the inverse f^{-1} of f.

(e) Find the domain and the range of f^{-1}.

(f) Graph f^{-1}.

15. Solve the inequality $(x+1)^2(x-2)(x-3)^3 \geq 0$.